Problem 17

$\psi = \{(a,b)|a, b \in \mathbf{Z}; b \neq 0\}$ The relation ~ is defined by (a,b) ~ (c,d) iff ad = bc.

~ is an equivalence relation because it is:

1. Reflexive: (a,b) ~ (a,b) since ab = ba.

2. Symmetric: if (a,b) ~ (c,d) then ad = bc, so cb = da and (c,d) ~ (a,b).

3. Transitive: if (a,b) ~ (c,d) and (c,d) ~ (e,f) then ad = bc and cf = de. Then adcf = bcde, if $c \neq 0$ then cancelling dc from each side gives: af = be. If c = 0 then a = 0 and e = o so af = be again. In both cases af = be so (a,b) ~ (e,f).

The Equivalence class for (1,2), $T_{(1,2)} = \{(a, 2a)|\ a \neq 0 \text{ and } a \in \mathbf{Z}\}$ this corresponds to the rational number .5

The Equivalence class for (3,4), $T_{(3,4)} = \{(3a, 4a)|\ a \neq 0 \text{ and } a \in \mathbf{Z}\}$ this coressponds to the rational number .75

The Equivalence class for (a,b), $T_{(a,b)} = \{(ax, bx)|\ x \neq 0 \text{ and } x \in \mathbf{Z}\}$ this coressponds the the rational number, one of whose representations is a/b.

This is the equivalence relation that shows us when two fractions of integers represent the same rational number. There is an equivalence class for every rational number.